Pre-empting Competitive Sales: Private Offers, Early Resolution, and Seller Information

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Abstract

A private offer can end a competitive sale before other buyers bid. We study a buyer who privately knows its value and benefit from early agreement, facing a seller with private information about the value of continuing to auction. Accepted offers must finance both the competition the parties forgo and the cost of unsuccessful attempts. Some buyers make probing offers that only sellers with weak continuation prospects accept; buyers with larger resolution benefits may pay more to secure acceptance in either state. A ban preserves access for later buyers and avoids offer costs, but sacrifices gains from early resolution. Its welfare effect depends on the surplus excluded buyers would have earned.

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1 Introduction

A seller preparing a competitive sale may receive a private offer to end the process early. Accepting gives the parties certainty, but prevents other buyers from bidding. A homeowner may sell before an announced bidding round; a business may accept an offer before its equipment auction. Consider also someone reselling a concert ticket: an offer today competes with the possibility of stronger demand close to the event. These settings share a decision about whether to agree now or preserve the opportunity for later competition.

We study this decision when the seller knows more about the alternative to early agreement than the buyer does. The buyer privately knows its value for the object and its benefit from resolving the purchase early. Making an offer is costly. The seller privately observes a state that affects its expected payoff from a subsequent English auction. The buyer chooses a binding price and, after rejection, remains free to bid in that auction. Its offer therefore both screens the seller's willingness to continue and conveys information about a future competitor.

The model explains why buyers make offers that may be rejected. A relatively low offer purchases early agreement only when the seller's continuation prospects are weak. We call this a *probe*. Paying more can secure acceptance even from a seller with strong prospects: a *knockout* offer. Buyers with little to gain from early resolution wait. Those with intermediate gains may probe, while buyers with larger gains pay for certain acceptance. The prices must be consistent with the buyer pools they attract, because those pools determine the seller's expected auction payoff after rejection.

Our first result identifies what finances early contracting. The seller and early buyer jointly sacrifice a nonnegative *competition wedge* by abandoning the auction. When the buyer's value exceeds the seller's fallback, this wedge comes from the possibility that at least two later buyers bid above that value. In every perfect Bayesian equilibrium, the resolution gains from accepted trades must cover this wedge and the cost of all offer attempts, including rejected ones. This result allows arbitrary price menus and mixed strategies.

We then construct equilibria with waiting and probing, and give sufficient conditions for knockout to coexist with them. Continuously distributed buyer values and resolution benefits determine who selects each action. We use locally passive beliefs to select prices at which the marginal accepting seller is indifferent, and separately show when the resulting on-path outcomes admit beliefs satisfying criterion D1. The two restrictions have different roles: D1 does not uniquely select these prices, and our existence results do not establish global uniqueness.

Finally, we compare discretion to accept early offers with a ban that leaves the terminal auction unchanged. The early buyer values the option to offer, and the seller accepts only when doing so is worthwhile given its information. Their agreement can nevertheless exclude a buyer who values the object more. One high late value is enough to create an allocative loss; two are needed to push the auction price above the early buyer's value. This difference between allocative value and the contracting parties' opportunity cost drives the welfare result. A ban saves offer costs and preserves late-buyer surplus, but removes gains from early resolution. Two examples with the same mean buyer value give opposite welfare rankings.

Housing provides a documented application. Norwegian *kupping* occurs before the ordinary bidding process (Finansdepartementet, 2021, p. 66); auction rules in Australia and New Zealand permit sales before the scheduled auction (NSW Fair Trading, nd; Real Estate Authority, nd). Pre-auction offers are also invited in sales of industrial equipment (Prestige Equipment, 2026); private acquisition offers may be conditional on a target ceasing to entertain competing bidders (Liu et al., 2023, pp. 7–8). Our results concern an English-auction continuation, which gives a tractable account of competition, price formation, and allocation. The ticket example illustrates the broader economic question; sequential enquiries and changing posted prices would require a different continuation model.

The closest procedural comparison is Grebe et al. (2016), where one buyer makes a binding Sell-It-Now offer and enters a second-price auction with a rival after rejection. Their risk-neutral benchmark has a publicly known seller value of zero, fixed participation, no attempt cost, and no direct resolution benefit; under their tie-breaking rule every offer is rejected. We study how private seller continuation prospects interact with voluntary costly offers and heterogeneous resolution benefits. Their working-paper version refers to Grebe's dissertation for a risk-aversion analysis (Grebe et al., 2015, fn. 7); our comparison concerns the published risk-neutral benchmark.

Seller-posted buyout prices can generate early trade through time sensitivity or risk aversion (Mathews, 2004; Gallien and Gupta, 2007; Budish and Takeyama, 2001; Mathews and Katzman, 2006). In Khezr and Menezes (2018), buyers accept an asking price or submit counteroffers; rejected counteroffers do not trigger an auction. Public pre-emptive bids can deter rival entry or information acquisition (Fishman, 1988; Cramton and Schwartz, 1991). Here the informed buyer privately chooses a binding price, screens the seller, and remains a competitor after rejection. That last feature makes the seller's continuation value depend on the information conveyed by the offer. Our welfare comparison concerns the option to terminate a fixed auction, whereas Bulow and Klemperer (2009) compare auctions with sequential negotiations.

Section 2 gives the model and financing result. Section 3 constructs probing and knockout equilibria. Section 4 studies the ban. Proofs follow in the paper's appendices. The Online Appendix studies further regimes, disclosure, risk aversion, and alternative sale protocols.

2 Private Offers before Competitive Sales

2.1 The offer and the auction

A seller owns one indivisible object. An early buyer privately observes its value $b \sim F$ on $[\underline{b}, \bar{b}]$ and benefit from early resolution $d_B \sim G$ on $[0, \bar{d}]$. The two draws are independent and have continuously differentiable positive densities f and g. Resolution benefits can reflect coordinating a linked transaction or avoiding costly delay; they need not covary with the buyer's match value. Write d for a realization of d_B.

The seller privately observes $s \in \{L, H\}$, independent of the buyer's type, with $\Pr(s = H) = \pi \in (0, 1)$. State s determines the distribution Q_s of the number N of late buyers, the seller's no-sale value $v_s \in (\underline{b}, \bar{b})$, and its cost of delay $d_{S,s} \geq 0$. Every late buyer draws independently from the same F as the early buyer. The distributions are common knowledge, but the buyer does not observe the seller's state. Participation is exogenous. We use stronger prospects for late participation as the running interpretation of H; a higher fallback or lower seller urgency can also raise the value of continuing.

The early buyer either waits or pays a cost $\kappa > 0$ to submit a private, binding price p. The seller observes the price and accepts or rejects. Acceptance ends the sale. Rejection leads to the same auction as waiting, with the early buyer still present. Late buyers arrive, the realized participants are observed, and all buyers compete in an English auction with independent private values. The seller is committed to the terminal rule: the object goes to the highest bidder if its value reaches v_s, at the larger of the second-highest bid and v_s; otherwise the seller keeps it. The threshold is fixed, although privately known at the offer stage. Truthful terminal bidding is optimal by the usual private-value argument: a bid changes whether the buyer wins, but not its payment conditional on winning against a given competing threshold.

Before the offer cost, early agreement at p gives the buyer $b - p + d_B$ and the seller p. Reaching the auction gives each party its auction payoff and costs the seller $d_{S,s}$. For convenience, subtract d_B from every outcome of a given buyer type: its normalized early payoff is $b - p$, and its normalized continuation payoff is auction surplus less d_B. The buyer additionally pays κ whenever it offers, whether accepted or rejected. All agents are risk neutral. The equilibrium concept is perfect Bayesian equilibrium (PBE). Except in the aggregate financing result, we construct pure strategies, with sellers accepting at indifference.

Let $Y_{1N} \geq Y_{2N}$ denote the two highest late values, setting a missing order statistic equal to $\underline{b}$. Conditional on (b, s), the early buyer's expected auction surplus and the seller's expected gross auction payoff are

$$u_s(b) = \mathbb{E}_s[\max\{b - \max\{v_s, Y_{1N}\}, 0\}], \tag{2.1}$$

$$R_s(b) = \mathbb{E}_s[P_s(b)], \qquad P_s(b) = \max\{v_s, Y_{2N}, \min\{b, Y_{1N}\}\}. \tag{2.2}$$

Here $\mathbb{E}_s$ averages over late participation and values. The seller's payoff includes v_s when there is no sale. Define the buyer's willingness to pay for ownership before resolution benefits, $w_s(b) = b - u_s(b)$, and the seller's net continuation value,

$$C_s(b) = R_s(b) - d_{S,s}, \qquad \bar{w}(b) = (1 - \pi)w_L(b) + \pi w_H(b). \tag{2.3}$$

These are conditional payoff calculations: they do not suppose that either party observes the other's private information.

2.2 The competition wedge and the financing of attempts

The seller and early buyer can divide b by selling early, before resolution benefits. Together they obtain $R_s(b) + u_s(b)$ in the auction. Their opportunity cost of early agreement is therefore

$$\Gamma_s(b) := R_s(b) - w_s(b) = \mathbb{E}_s[\max\{\max\{v_s, Y_{2N}\} - b, 0\}] \geq 0. \tag{2.4}$$

Appendix A proves the identity. If $b \geq v_s$, the wedge is positive precisely when there is positive probability of two late values above b. For example, take $b = 100$ and a lower seller fallback. One late buyer with value 140 wins at 100, leaving the seller and early buyer jointly with 100. Two late values of 140 and 120 instead give the seller 120. In the latter realization, the auction produces 20 more than these two parties can divide through an early sale.

For fixed (b, s, d), and before the attempt cost, the price must lie between the seller's net continuation value and the buyer's willingness to pay including resolution. Thus the bilateral feasibility condition is

$$C_s(b) \leq p \leq w_s(b) + d \quad \iff \quad d + d_{S,s} \geq \Gamma_s(b). \tag{2.5}$$

When $b < v_s$, the wedge includes the value of retaining the object. When $b \geq v_s$, it comes entirely from late competition. Private information prevents the parties from simply choosing a separate price for every (b, s). The next result gives the corresponding restriction on actual equilibrium trade.

Proposition 2.1 (Aggregate financing of early offers)**.** *In every PBE of the one-offer game, allowing arbitrary price menus and mixed strategies, let I be the event that an offer is made and $A_c \subseteq I$ the event that it is accepted. Then*

$$\mathbb{E}[\{d_B + d_{S,s} - \Gamma_s(b)\}\mathbf{1}_{A_c}] \geq \kappa \Pr(I). \tag{2.6}$$

The conclusion also holds at $\kappa = 0$.

Proof. Measure each party's gain relative to waiting. Buyer optimality gives $\mathbb{E}[g_B \mid b, d] \geq 0$. At every on-path (s, p), seller optimality and Bayes' rule give $\mathbb{E}[g_S \mid s, p] \geq 0$: acceptance is chosen only if its conditional expected gain is nonnegative, while rejection gives zero. After acceptance, prices cancel and $g_B + g_S = d + d_{S,s} - \Gamma_s(b) - \kappa$. After rejection the sum is $-\kappa$, and after waiting it is zero. Taking expectations proves the inequality. The argument permits mixing and conditions on each party's own information. □

Accepted trades must pay for unsuccessful attempts as well as for the competition forgone. If acceptance has positive probability, their average net resolution gain is at least $\kappa \Pr(I)/\Pr(A_c)$. With no resolution gains and positive attempt costs, no buyer offers. This restriction does not depend on a particular selection of prices or off-path beliefs.

2.3 Private continuation values

The equilibrium construction uses the following ordering. Appendix A.1 derives it from participation prospects, the seller's fallback, and seller urgency.

Assumption 1 (Ordered continuation values)**.** The functions w_s are continuous and strictly increasing; C_s are continuous, nondecreasing, and strictly increasing on a top interval, with $C_L(\underline{b}) > 0$. For every b, $w_L(b) \leq w_H(b)$ and $C_L(b) < C_H(b)$. The gap $w_H - w_L$ is weakly increasing, and Γ_L is weakly decreasing.

After observing p, the seller updates its belief about the early buyer's value. Let $\mu_B(\cdot \mid p)$ denote this posterior. It is common across seller states because the buyer cannot condition its offer on s, which is independent of its type. Seller s accepts exactly when

$$p \geq \mathcal{C}_s(p) := \int C_s(b)\, d\mu_B(b \mid p). \tag{2.7}$$

Pointwise ordering implies $\mathcal{C}_L(p) < \mathcal{C}_H(p)$, even if the ranges of the two continuation functions overlap. Acceptance is therefore nested: neither state accepts, only L accepts, or both accept. Under pure responses, two used prices with the same accepting states cannot differ, since every buyer would prefer the lower price. An offer rejected in both states loses κ and is dominated by waiting. At most two positive prices remain relevant.

Private seller information makes rejection possible without making the attempt pointless. If the buyer instead observed s before offering, no costly offer known to be rejected could be used in a pure-strategy PBE: waiting gives the same continuation at lower cost. This observation concerns a different information structure, rather than the conditional payoffs in (2.1)–(2.5).

3 Probing and Certain Acceptance

3.1 Buyer selection and prices

Denote waiting by W, a probing offer q_L by P, and a knockout offer q_H by K. Their incremental payoffs relative to waiting are

$$\begin{aligned} V_0(b,d) &= 0, \\ V_L(b,d) &= (1-\pi)[w_L(b)+d-q_L]-\kappa, \\ V_H(b,d) &= \bar{w}(b)+d-q_H-\kappa. \end{aligned} \tag{3.1}$$

Probing purchases resolution only in state L; knockout purchases it in both states. Hence the payoff from knockout rises faster with d. The waiting–probing and probing–knockout cutoffs are

$$D_L(b;q_L) = q_L - w_L(b) + \frac{\kappa}{1-\pi}, \tag{3.2}$$

$$D_H(b;q_L,q_H) = \frac{q_H-(1-\pi)q_L}{\pi} - w_H(b). \tag{3.3}$$

For given prices with $D_L < D_H$, the buyer waits below D_L, probes between the cutoffs, and chooses knockout above D_H. Both cutoffs decline with value. A higher-value buyer needs a smaller resolution benefit to justify either switch.

Prices also affect the seller's inference. We select *boundary prices*: a probe makes L indifferent, and knockout makes H indifferent. Local passivity of beliefs after small undercuts justifies this selection. If an accepting state earned positive rent at a used price, a sufficiently small undercut that preserved its posterior would retain acceptance and benefit the buyer. Appendix C.1 states the belief restriction precisely. It does not rule out multiple boundary-price fixed points.

3.2 Probing with acceptance and rejection

For a candidate probing price q, buyers with $d \geq D_L(b;q)$ offer. With the convention $G(x) = 0$ below zero and $G(x) = 1$ above $\bar{d}$, seller L's posterior continuation value is

$$T_L(q) = \frac{\int C_L(b)[1 - G(D_L(b;q))]f(b)\,db}{\int [1 - G(D_L(b;q))]f(b)\,db}. \tag{3.4}$$

Unspecified value integrals run over $[\underline{b}, \bar{b}]$. A fixed point $q_L = T_L(q_L)$ makes L indifferent, while H prefers to continue under the same posterior. Define the top-type entry thresholds and their difference by

$$\begin{aligned} \bar{D}_L &= C_L(\bar{b}) - w_L(\bar{b}) + \frac{\kappa}{1-\pi}, \\ \bar{D}_H &= C_H(\bar{b}) - \bar{w}(\bar{b}) + \kappa, \qquad \Delta_D = \bar{D}_H - \bar{D}_L. \end{aligned} \tag{3.5}$$

They measure the resolution benefit needed to offer when an unused price is attributed to the highest-value buyer. This belief maximizes the seller's continuation value and is therefore the most demanding one for an offer to satisfy.

Proposition 3.1 (Probing equilibrium)**.** *Under Assumption 1, a PBE in which every buyer waits exists if and only if*

$$\bar{d} \leq \min\{\bar{D}_L, \bar{D}_H\}. \tag{3.6}$$

Suppose instead that

$$\Delta_D > 0, \qquad \bar{D}_L < \bar{d} \leq \bar{D}_L + \Delta_D/\pi, \qquad C_L(\underline{b}) - w_L(\underline{b}) + \kappa/(1-\pi) > 0. \tag{3.7}$$

Then T_L has an interior fixed point $q_L \in (C_L(\underline{b}), C_L(\bar{b}))$. There is a PBE in which the buyer waits below $D_L(b;q_L)$ and offers q_L otherwise; L accepts and H rejects. Waiting, rejected offers, and accepted offers each have positive probability. If in addition $\bar{D}_L \geq 0$, this on-path outcome admits a D1-compatible belief completion in the contingent-plan representation of Appendix C.

The price map crosses the diagonal because its posterior mean exceeds the lowest continuation value and lies below the highest. The lower bound in (3.7) makes offering worthwhile for some buyers; the upper bound ensures that even the highest-value, most urgent buyer cannot gain by paying for acceptance in both states. Thus the proof must check deviations to knockout even though knockout is not used. Appendix B.2 gives the complete argument.

For a simple participation example, let $F = U[0,1]$, $v_L = v_H = .4$, $d_{S,L} = d_{S,H} = 0$, $(n_L, n_H) = (2,6)$, $\pi = .5$, and $\kappa = .02$. Equation (A.2) gives $\bar{D}_L = .04$ and $\bar{D}_H \simeq 0.10469$.

For example, $\bar{d} = .1$ satisfies the probing bounds, and the resulting on-path outcome admits D1 beliefs. Private information about later participation alone therefore suffices for accepted and rejected attempts; seller preferences need not differ across states.

3.3 Paying for certain acceptance

The gain from choosing knockout rather than probing is

$$V_H(b,d) - V_L(b,d) = \pi[w_H(b) + d] - q_H + (1-\pi)q_L.$$

A higher resolution benefit can justify the extra payment for acceptance in state H. This comparison explains buyer choice at given prices. Equilibrium additionally requires that the buyers choosing each price generate the seller expectations that sustain it.

For $q = (q_L, q_H)$ with $0 < D_L(b) < D_H(b) < \bar{d}$, define the conditional action masses and posterior densities by

$$\begin{aligned} m_W(b;q) &= G(D_L(b)), \\ m_P(b;q) &= G(D_H(b)) - G(D_L(b)), \\ m_K(b;q) &= 1 - G(D_H(b)), \end{aligned} \tag{3.8}$$

$$M_a(q) = \int m_a(b;q) f(b)\, db, \qquad h_a(b;q) = \frac{m_a(b;q)f(b)}{M_a(q)}. \tag{3.9}$$

The price map is

$$T_P(q) = \int C_L(b) h_P(b;q)\, db, \qquad T_K(q) = \int C_H(b) h_K(b;q)\, db. \tag{3.10}$$

At $q = T(q)$, the seller is indifferent in L after probing and in H after knockout. In state L, knockout leaves a rent: the higher price is needed to secure acceptance in the other state.

Proposition 3.2 (Probing and knockout)**.** *Suppose Assumption 1 and the price-rectangle conditions in Appendix B.3 hold. Then T has a fixed point $q_L < q_H$ supporting a PBE with*

$$p(b,d) = \begin{cases} \varnothing, & d < D_L(b), \\ q_L, & D_L(b) \le d < D_H(b), \\ q_H, & d \ge D_H(b). \end{cases} \tag{3.11}$$

Seller L accepts both prices; seller H accepts only q_H. All three actions have positive probability at every buyer value. The on-path outcome admits both a locally passive PBE completion and a D1-compatible completion in the contingent-plan representation, which need not use the same off-path beliefs.

The sufficient conditions place the price map inside a rectangle on which both cutoffs are interior and the map points inward at each relevant face. Continuity then yields consistent prices. The proof checks seller acceptance and buyer deviations at every unused price. Requiring all three actions at every value is stronger than requiring positive aggregate action masses; the conditions constrain equilibrium prices as well as the support of resolution benefits.

The observable probabilities are M_W for no offer, πM_P for rejection, and $(1-\pi)M_P + M_K$ for successful pre-emption. Knockout adds certain acceptance to a mechanism that already generates attempts and rejections through probing. The Online Appendix also characterizes equilibria in which all submitted offers are accepted.

4 Restricting Pre-emption: Allocation and Welfare

Compare seller discretion, denoted D, with a ban on early agreement, denoted B. The ban sends every buyer to the same terminal auction, holding participation, values, and costs fixed. This comparison applies to sales with an identifiable early-offer stage and an enforceable restriction on early agreement.

The early buyer weakly prefers discretion because waiting remains available. The seller also weakly prefers it at the offer stage, accepting only when its conditional expected payoff warrants acceptance. In the boundary-price equilibrium with positive knockout mass, its ex ante gain is strictly positive: a knockout offer pays for state H even when the seller is in L. Later buyers weakly prefer the ban, which restores their opportunity to win. Appendix D derives these incidence results.

The allocative benefit of preserving the auction after a proposed early sale to value b is

$$L_s(b) = \mathbb{E}_s[\max\{\max\{v_s, Y_{1N}\} - b, 0\}]. \tag{4.1}$$

Count v_s as the social value of retaining the object, rather than a later transfer; count d_B and $d_{S,s}$ at their monetary utility values and κ as a real effort cost. The highest late value determines whether the auction improves allocation. By contrast, the second-highest late value determines the contracting parties' wedge. Their difference is the winning late buyer's expected surplus,

$$\Lambda_s(b) = \mathbb{E}_s[\max\{Y_{1N} - \max\{v_s, b, Y_{2N}\}, 0\}], \qquad L_s(b) = \Gamma_s(b) + \Lambda_s(b). \tag{4.2}$$

The equality holds pointwise before taking expectations. Early agreement eliminates both the opportunity cost borne by its participants and the surplus left to an excluded buyer.

Proposition 4.1 (Welfare effect of a ban)**.** *Holding the terminal environment fixed, the welfare difference is*

$$W^B - W^D = \mathbb{E}[\kappa \mathbf{1}_I + \mathbf{1}_{A_c}\{L_s(b) - d_B - d_{S,s}\}]. \tag{4.3}$$

A ban raises surplus precisely when the allocative gains and avoided offer costs exceed the resolution gains from accepted trades.

Proof. After a rejected offer, both regimes reach the same auction, but the ban saves κ. After an accepted offer, it also replaces an allocation to b by one worth $\max\{v_s, b, Y_{1N}\}$, while forgoing d_B and incurring $d_{S,s}$. No offer leaves the regimes identical. Averaging and cancelling transfers gives (4.3). □

The financing result and the welfare result concern different surpluses. The private gains from accepted offers must cover rejected attempts in equilibrium. Even so, the parties need not internalize $\Lambda_s(b)$. A larger private benefit from resolution makes early contracting easier to sustain, but a larger excluded late-buyer surplus makes preserving competition more valuable socially.

Table 4.1 gives two examples with identical parameters except for the common value distribution of early and late buyers. Both distributions have mean .5. The first is concentrated around the mean; the second adds probability near the upper end while preserving that mean. In both cases, buyers wait, probe, or choose knockout. Appendix D.2 specifies the distributions and supplies bounds certifying equilibrium existence and the welfare signs.

Discretion raises welfare under F^S, while the ban raises it under F^T. A single exceptional late value can increase the allocative value of waiting without raising the auction price proportionately. Excluded late-buyer surplus per accepted early sale rises from .01909 to .03689 between the examples. Early-sale incidence falls as prices and buyer pools adjust to the new distribution.

The ban keeps late participation fixed. If cancellation also saves preparation costs for late buyers, those savings must be subtracted from its measured benefit. Conversely,

Table 4.1: The Welfare Effect of a Ban with Mean-Matched Values

	Concentrated F^S	Upper tail F^T
Rejected attempt	.05631	.06674
Successful pre-emption	.15435	.06839
Avoided attempt costs	.00421	.00270
Allocative gain	.00388	.00276
Forgone resolution gains	.01162	.00516
$W^B - W^D$	-.00353	.00030

Notes: The first two rows are probabilities; welfare entries are per object. Seller delay costs are zero. Reported point estimates are rounded numerical approximations; interval bounds in Appendix D.2 establish the signs, rather than all displayed decimal places.

allowing participation to respond to expected cancellation would require a model of entry. These margins matter for applications, but are distinct from the allocation–resolution tradeoff in (4.3).

5 Conclusion

A private offer before a competitive sale purchases early resolution at the cost of possible competition. Seller information makes this purchase uncertain: a probing price is accepted when continuation prospects are weak and rejected when they are strong. Buyers with sufficiently large resolution benefits may instead pay for acceptance in both states. The offer also reveals information about a buyer who returns after rejection, so prices and continuation expectations must be jointly consistent.

The same mechanism creates a policy tradeoff. The seller and early buyer internalize the auction revenue they forgo, but not all the surplus available to later buyers. Restricting pre-emption protects that surplus and saves attempt costs while removing gains from early agreement. English auctions make these components explicit. Studying other competitive-sale and search mechanisms would clarify how the tradeoff changes when the continuation process differs.

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A Auction Payoffs and Sources of Seller Information

Let $\phi_s(x) = \mathbb{E}_{Q_s}[x^N]$, with $x^0 = 1$, be the probability-generating function of late participation. Then $\phi_s(F(t))$ is the probability that every late value is below t. Integration of the early buyer's winning probability gives

$$w_s(b) = \begin{cases} b, & b \le v_s, \\ v_s + \int_{v_s}^{b} [1 - \phi_s(F(t))]\, dt, & b > v_s. \end{cases} \tag{A.1}$$

The no-sale probability is $\mathbf{1}\{b < v_s\}\phi_s(F(v_s))$ conditional on (b, s). The unconditional probability in state s is $F(v_s)\phi_s(F(v_s))$; stronger participation lowers it, while a higher fallback raises it.

To prove (2.4), set $Z = \max\{v_s, Y_{2N}\}$ and $M = \min\{b, \max\{v_s, Y_{1N}\}\}$. The terminal price is $\max\{Z, M\}$, and $w_s(b) = \mathbb{E}_s[M]$. If $Z \le b$, then $Z \le M$; otherwise $M = b$. Thus the price less M equals $\max\{Z - b, 0\}$ in both cases. The wedge is weakly decreasing in b, and $\Gamma_s(\bar{b}) = 0$, so $C_s(\bar{b}) = w_s(\bar{b}) - d_{S,s}$.

For a deterministic count $n \ge 1$ and $F = U[0, 1]$, direct integration gives

$$\begin{aligned} w_{n,v}(b) &= \begin{cases} b, & b \le v, \\ b - \dfrac{b^{n+1} - v^{n+1}}{n+1}, & b > v, \end{cases} \\ R_{n,v}(b) &= \begin{cases} \dfrac{n-1}{n+1} + v^n - \dfrac{n-1}{n+1} v^{n+1}, & b \le v, \\ \dfrac{n-1}{n+1} + b^n - \dfrac{n}{n+1} b^{n+1} + \dfrac{v^{n+1}}{n+1}, & b > v. \end{cases} \end{aligned} \tag{A.2}$$

A.1 Sources of seller information

Suppose $Q_s(N \ge 1) > 0$ and $d_{S,L} < R_L(\underline{b})$. The following three structures satisfy Assumption 1. Counts are finite almost surely, and the formulas below apply wherever the displayed derivatives exist; they also establish the corresponding monotonicity at kinks.

Participation. Hold $v_s = v_0$ and $d_{S,s} = d_S$ fixed. Let Q_H first-order stochastically dominate Q_L, with a strict tail inequality $\Pr_H(N \ge k) > \Pr_L(N \ge k)$ for some $k \ge 2$. Monotone coupling gives $w_H \ge w_L$ and $C_H \ge C_L$. For $b > v_0$,

$$w_H(b) - w_L(b) = \int_{v_0}^{b} [\phi_L(F(t)) - \phi_H(F(t))]\, dt,$$

which is strictly increasing. The extra tail condition makes the seller inequality strict even below the fallback; its proof follows below.

Fallback. Hold Q and d_S fixed, and let $v_H > v_L$. Then

$$C_H(b) - C_L(b) = \int_{v_L}^{v_H} \Pr(\max\{Y_{2N}, \min\{b, Y_{1N}\}\} \le t)\ dt > 0.$$

The integrand is weakly decreasing in b. The willingness gap is zero below v_L and equals $\int_{v_L}^{\min\{b,v_H\}} \phi(F(t))\, dt$ above it, increasing to a plateau at v_H.

Urgency. Hold Q and v fixed, and let $d_{S,L} > d_{S,H} \ge 0$. Then $w_H = w_L$ and $C_H - C_L = d_{S,L} - d_{S,H} > 0$. Urgency changes the seller's value of continuing without changing the buyer's auction payoff. Participation and fallback information can change both.

These changes can also be combined in the same direction: stronger participation, a higher fallback, and lower seller delay costs in H. For $v_H \ge v_L$, the willingness difference is

$$w_H(b) - w_L(b) = \begin{cases} 0, & b \le v_L, \\ \displaystyle\int_{v_L}^{b} \phi_L(F(t))\, dt, & v_L < b \le v_H, \\ \displaystyle\int_{v_L}^{v_H} \phi_L(F(t))\, dt + \int_{v_H}^{b} [\phi_L(F(t)) - \phi_H(F(t))]\, dt, & b > v_H. \end{cases} \tag{A.3}$$

This is weakly increasing. In each state,

$$R_s'(b) = \begin{cases} 0, & b < v_s, \\ [1 - F(b)]\phi_s'(F(b)) > 0, & v_s < b < \bar{b}. \end{cases} \tag{A.4}$$

Equations (A.1) and (A.4) give regularity and monotonicity. The wedge formula gives the remaining selection shape. Any of the strict state changes just specified makes $C_H > C_L$ everywhere. Thus the same equilibrium argument covers all three sources of seller information without requiring that they have identical effects on buyer selection.

Under the aligned primitive changes, monotone coupling also gives $\Gamma_H(b) \ge \Gamma_L(b)$. Since $d_{S,H} \le d_{S,L}$, the bilateral resolution thresholds $\Gamma_s(b) - d_{S,s}$ are ordered. Condition (2.5) therefore holds in neither state, in L alone, or in both states as d rises. A strict inequality permits strict gains for both parties. This is a conditional feasibility statement, not an equilibrium price rule under private information.

Strict ordering under participation

For $x \in (0,1)$, let $a_n(x)$ be the probability that at least two of n draws exceed quantile x. Since $a_0 = a_1 = 0$ and $a_n - a_{n-1} = (n-1)(1-x)^2 x^{n-2} > 0$ for $n \geq 2$, the tail-sum identity gives

$$\Pr_H(Y_{2N} > t) - \Pr_L(Y_{2N} > t) = \sum_{k=2}^{\infty}(k-1)[1-F(t)]^2 F(t)^{k-2}\{\Pr_H(N \geq k) - \Pr_L(N \geq k)\}.$$

For $v_0 < t < \bar{b}$, this is strictly positive under the stated tail condition. If $b \leq v_0$, integrating above v_0 gives $C_H(b) > C_L(b)$. If $v_0 < b < \bar{b}$, integrate above b, noting that the difference below b is weakly positive. At $b = \bar{b}$, a nontrivial count shift strictly increases the probability that at least one late value exceeds each $t \in (v_0, \bar{b})$. This establishes strictness at the endpoint as well. A shift only from zero to one late buyer cannot produce strict ordering below the fallback, because the terminal price there remains v_0.

B Proofs of the Equilibrium Results

B.1 Prices and action support

Nested acceptance limits the number of used prices. The following condition also ensures a positive mass of waiting buyers:

$$\chi_L = C_L(\underline{b}) - w_L(\underline{b}) + \frac{\kappa}{1-\pi} > 0, \qquad \chi_H = C_H(\underline{b}) - \bar{w}(\underline{b}) + \kappa > 0. \tag{B.1}$$

These inequalities require the least-valued patient buyer to prefer waiting even at the lowest possible accepted price. Proposition 3.1 uses the first inequality; Proposition 3.2 instead imposes the stronger cutoff interiority below.

Lemma B.1 (On-path prices and action support)**.** *Suppose $\kappa > 0$. Restrict attention to pure-strategy PBE, with seller indifference resolved in favor of acceptance. All on-path probing prices then coincide almost surely, and all on-path knockout prices coincide almost surely. No offer rejected in both states is submitted with positive probability. Under* (B.1)*, every such PBE has positive waiting mass. Thus, denoting the unique probing and knockout price actions by P and K when present, the action support is $\{W\}$, $\{W,P\}$, $\{W,K\}$, or $\{W,P,K\}$.*

Proof. The common posterior and $C_L < C_H$ imply nested acceptance. Any price rejected by both states is dominated by waiting. For two prices inducing the same nonempty acceptance pattern, every buyer strictly prefers the lower; thus the price is constant almost surely within that pattern, even if the candidate price rule has a continuous range. Seller optimality gives $q_L \geq C_L(\underline{b})$ and $q_H \geq C_H(\underline{b})$. At $(\underline{b}, 0)$, offer gains are bounded above by

$-(1-\pi)\chi_L$ and $-\chi_H$, respectively. Continuity and positive type densities give a positive neighborhood of waiting types. The two price bounds then leave exactly the four possible supports stated. □

B.2 Probing

Proof of Proposition 3.1. For sufficiency of the no-early-offer condition, assign every off-path price to $b = \bar{b}$. The cheapest prices inducing L-only and both-state acceptance are then $C_L(\bar{b})$ and $C_H(\bar{b})$. Their payoffs are increasing in (b, d), so the two inequalities in (3.6) deter every buyer. The same highest-type posterior survives D1 in the contingent-plan representation: under every rationalizable plan with positive acceptance probability, the gain from the unused price is weakly increasing in urgency and in the state-contingent auction willingness $w_s(b)$, while rejection in both states loses κ for every type. Hence no other type can D1-dominate $(\bar{b}, \bar{d})$.

For necessity, first suppose $\bar{d} > \bar{D}_H$. A price just above $C_H(\bar{b})$ is accepted in both states under every posterior and is profitable for types near $(\bar{b}, \bar{d})$. If instead $\bar{d} > \bar{D}_L$, a price just above $C_L(\bar{b})$ is accepted by seller L under every posterior and its L-only payoff is positive for the top type. If seller H also accepts under the induced posterior, this only raises that type's payoff: at the limiting price,

$$w_H(\bar{b}) + \bar{d} - C_L(\bar{b}) = w_H(\bar{b}) - w_L(\bar{b}) + \bar{d} + d_{S,L} > 0,$$

where $R_L(\bar{b}) = w_L(\bar{b})$. Thus violation of either inequality creates a profitable deviation even when continuation-value supports overlap.

Now impose (3.7). Its strict lower bound rules out no-early-offer PBE. On $[C_L(\underline{b}), C_L(\bar{b})]$, the probing map is continuous wherever its denominator is positive. At the lower endpoint its posterior mean is strictly above $C_L(\underline{b})$: indeed, $\bar{d} > \bar{D}_L$ and $C_L(\underline{b}) < C_L(\bar{b})$ imply

$$D_L(\bar{b}; C_L(\underline{b})) < \bar{D}_L < \bar{d},$$

so a positive interval of high values on which $C_L(b) > C_L(\underline{b})$ probes. At the upper endpoint, $D_L(\bar{b}; C_L(\bar{b})) = \bar{D}_L < \bar{d}$, so a positive interval of high-b types probes and the posterior mean is strictly below $C_L(\bar{b})$. The intermediate value theorem gives an interior fixed point q_L. At that point the cutoff $D_L(b; q_L)$ implements the stated buyer strategy, Bayes' rule makes seller L indifferent at q_L, and seller H rejects because, under the same probe posterior μ_B,

$$\int C_H(b)\, d\mu_B(b) > \int C_L(b)\, d\mu_B(b) = q_L.$$

After prices in a sufficiently small left neighborhood of q_L, retain the on-path probing posterior. Seller L's continuation value is then q_L, so an undercut is rejected by both seller states. Assign every remaining unused price to $b = \bar{b}$. A lower price is rejected, while a higher price that only seller L accepts is dominated by q_L. The cheapest price accepted in both states is $C_H(\bar{b})$. Relative to probing, its payoff satisfies

$$\begin{aligned}
&\bar{w}(b) + d - C_H(\bar{b}) - \kappa - \{(1-\pi)[w_L(b) + d - q_L] - \kappa\} \\
&\quad = \pi[w_H(b) + d] + (1-\pi)q_L - C_H(\bar{b}) \\
&\quad \leq \pi[w_H(\bar{b}) + \bar{d}] + (1-\pi)C_L(\bar{b}) - C_H(\bar{b}) \leq 0,
\end{aligned}$$

where the final inequality is equivalent to the upper bound on $\bar{d}$ in (3.7). Thus no knockout deviation gains, including for a type that waits, since a positive knockout payoff would imply a positive probing payoff.

The final condition in (3.7) gives $D_L(\underline{b}; q_L) > 0$, while $\bar{d} > \bar{D}_L > D_L(\bar{b}; q_L)$. Both waiting and probing therefore have positive probability. Since both continuation states have positive probability, probing generates both rejection and acceptance with positive probability.

□

B.3 The full menu

Boundary-price fixed points lie in

$$\mathcal{Q} = [C_L(\underline{b}), C_L(\bar{b})] \times [C_H(\underline{b}), C_H(\bar{b})]. \tag{B.2}$$

Let $\mathcal{Q}_0 = [\ell_L, u_L] \times [\ell_H, u_H] \subseteq \mathcal{Q}$, with both intervals nondegenerate. The price-rectangle conditions in Proposition 3.2 require, for every $q \in \mathcal{Q}_0$ and every b,

$$0 < D_L(b; q_L) < D_H(b; q) < \bar{d}, \tag{B.3}$$

and the inward face inequalities

$$\begin{aligned}
&T_P(\ell_L, q_H) > \ell_L, \qquad && T_P(u_L, q_H) < u_L \qquad && \text{for all } q_H \in [\ell_H, u_H], \\
&T_K(q_L, \ell_H) > \ell_H, \qquad && T_K(q_L, u_H) < u_H \qquad && \text{for all } q_L \in [\ell_L, u_L].
\end{aligned} \tag{B.4}$$

For the deviation comparisons define

$$\widetilde{D}_H(b; q_H) = q_H - \bar{w}(b) + \kappa, \tag{B.5}$$

$$Z(b; q) = q_H - q_L - \pi[w_H(b) - w_L(b)] - \frac{\pi\kappa}{1-\pi}. \tag{B.6}$$

These give

$$D_H - D_L = Z/\pi, \quad \widetilde{D}_H - D_L = Z, \quad D_H - \widetilde{D}_H = (1-\pi)Z/\pi. \tag{B.7}$$

Thus (B.3) implies $Z > 0$. By the monotonicity of w_L, w_H and $w_H - w_L$, interiority is equivalent to the three endpoint checks

$$\begin{aligned} \ell_L - w_L(\bar{b}) + \frac{\kappa}{1-\pi} &> 0, \\ \frac{u_H - (1-\pi)\ell_L}{\pi} - w_H(\underline{b}) &< \bar{d}, \\ \ell_H - u_L - \pi\Big[w_H(\bar{b}) - w_L(\bar{b})\Big] - \frac{\pi}{1-\pi}\kappa &> 0. \end{aligned} \tag{B.8}$$

These inequalities ensure all three action intervals at every match value, uniformly over $\mathcal{Q}_0$. Seller acceptance remains nested by the common-posterior ordering in Section 2.

Proof of Proposition 3.2. For every $q \in \mathcal{Q}_0$, condition (B.3) and the positive density of G imply that all three actions have positive probability for every b. The conditional action probabilities in (3.8) and their integrals $M_a(q)$ are continuous in q. Since each $M_a(q) > 0$, the posterior densities in (3.9) and the price map T are continuous as well. Let $\Psi(q) = T(q) - q$. Condition (B.4) makes the first coordinate of Ψ positive and negative on the lower and upper q_L-faces, respectively, and does the same for the second coordinate on the corresponding q_H-faces. The Poincaré–Miranda theorem therefore gives a zero of Ψ in $\mathcal{Q}_0$; the strict face inequalities put it in the interior.

Fix such a point. At q_L, seller L's posterior continuation value is

$$\int C_L(b) h_P(b; q)\, db = q_L,$$

so it accepts. Seller H rejects because

$$\int C_H(b) h_P(b; q)\, db > \int C_L(b) h_P(b; q)\, db = q_L.$$

At q_H, seller H accepts at equality, while seller L accepts strictly because

$$\int C_L(b) h_K(b; q)\, db < \int C_H(b) h_K(b; q)\, db = q_H.$$

Finally, $Z(b; q) > 0$ implies $q_H - q_L > \pi[w_H(b) - w_L(b)] + \pi\kappa/(1-\pi) > 0$, so $q_L < q_H$.

For each b, equation (B.7) and $Z(b; q) > 0$ imply

$$D_L(b) < \widetilde{D}_H(b) < D_H(b).$$

Below $D_L(b)$, both submitted-offer payoffs are negative. Between $D_L(b)$ and $\widetilde{D}_H(b)$, only probing is profitable. Between $\widetilde{D}_H(b)$ and $D_H(b)$, both offers are profitable and probing yields the larger payoff. Above $D_H(b)$, knockout yields the larger payoff. These comparisons prove (3.11).

It remains to rule out off-path deviations. First consider sufficiently small, disjoint left neighborhoods of the two on-path prices. After an undercut of q_L, retain the on-path probing posterior. Seller L's continuation value is then $q_L > p$, so both states reject. After an undercut of q_H, retain the on-path knockout posterior. Seller H's continuation value is $q_H > p$, so H rejects; if seller L accepts, the deviation induces the same L-only response as the strictly cheaper on-path probe q_L. No local undercut is profitable, and these posteriors converge to their on-path counterparts as required by Assumption 2.

At every remaining off-path price, concentrate the posterior on $b = \bar{b}$. A price below $C_L(\bar{b})$ is rejected by both sellers, a price in $[C_L(\bar{b}), C_H(\bar{b}))$ is accepted only by seller L, and a price at least $C_H(\bar{b})$ is accepted by both. A rejected deviation yields $-\kappa$. Since

$$q_L = \int C_L(b) h_P(b; q)\, db \le C_L(\bar{b}),$$

any off-path price accepted only in state L is weakly more expensive than probing. Likewise,

$$q_H = \int C_H(b) h_K(b; q)\, db \le C_H(\bar{b}),$$

so any off-path price accepted in both states is weakly more expensive than knockout. No buyer can improve on its best on-path action. Seller actions are sequentially rational by (2.7), completing the proof.

The D1-compatible completion is established in Appendix C. □

C Price Selection and Refinement

C.1 Local passivity

Assumption 2 (Locally passive beliefs)**.** For price selection, let $q > 0$ be an active offer and let $\mu_B(\cdot \mid q)$ be the seller's on-path posterior over the buyer's match value. After unused prices p approaching q from below, the posterior is locally passive:

$$\mu_B(\cdot \mid p) \Rightarrow \mu_B(\cdot \mid q) \qquad \text{as } p \uparrow q,$$

where $\Rightarrow$ denotes weak convergence.

Assumption 2 implies that every active price leaves the seller state at the acceptance margin indifferent. To see this, suppose that state instead earned strictly positive expected rent at an active price q. For all sufficiently close undercuts $p < q$, local passivity would keep that state's expected continuation payoff below p, so it would still accept. A probing undercut would retain acceptance by seller L and rejection by seller H; a knockout undercut would retain acceptance by both states. Any buyer using q could then induce the same seller responses at the lower price p, a profitable deviation. Hence the probing price satisfies $q_L = \mathcal{C}_L(q_L)$, while the knockout price satisfies $q_H = \mathcal{C}_H(q_H)$.

Local passivity selects binding participation constraints for the marginal accepting states. The following refinement test asks instead which buyer types can reasonably be inferred after deviations. We apply criterion D1 in a contingent-plan representation (Cho and Kreps, 1987; Banks and Sobel, 1987). A D1-compatible completion of the reported outcome need not retain the locally passive beliefs used to construct it.

C.2 The contingent-plan test

Lemma C.1 (Contingent-plan representation for criterion D1)**.** *Suppose the buyer does not observe s, takes no action after the seller responds, and the distribution of s is independent of buyer type. After any price p, the one-offer game is outcome- and payoff-equivalent to a signaling game in which the seller chooses a contingent acceptance plan $a = (a_L, a_H) \in [0,1]^2$. Criterion D1 in this representation is the strict-gain set-inclusion test used below.*

Proof. Let μ be the posterior over buyer type after p, and let $\lambda_s > 0$ be the prior probability of state s. The seller's expected payoff from plan a is

$$\sum_{s \in \{L,H\}} \lambda_s \left\{ a_s p + (1 - a_s) \int C_s(b)\, d\mu(b,d) \right\}.$$

This objective separates across states. A plan is therefore a best response if and only if each a_s is optimal at the corresponding seller information set (p, s) in the original game. The buyer's expected payoff from the same acceptance probabilities is unchanged. Moreover, independence and the buyer's inability to condition its offer on s imply $\mu(\cdot \mid p, s) = \mu(\cdot \mid p)$: one buyer tremble generates the same posterior in both seller states.

The receiver responses in the mixed-best-response formulation of D1 are therefore precisely these rationalizable contingent plans. Formally, let $\mathcal{A}(p)$ be the union, over posteriors μ, of the seller's best-response plans after p. For buyer type $\theta = (b, d)$, let $U^*(\theta)$ denote its equilibrium payoff and let $u_\theta(p, a)$ denote its payoff from deviating to price p when the seller uses plan a. Define

$$\mathcal{D}(\theta, p) = \{a \in \mathcal{A}(p) : u_\theta(p, a) > U^*(\theta)\},$$
$$\mathcal{D}^0(\theta, p) = \{a \in \mathcal{A}(p) : u_\theta(p, a) = U^*(\theta)\}.$$

The strict-gain D1 rule deletes θ if another type θ' satisfies

$$\mathcal{D}(\theta, p) \cup \mathcal{D}^0(\theta, p) \subsetneq \mathcal{D}(\theta', p).$$

Thus every rationalizable plan that makes the deleted type weakly willing to deviate makes the dominating type strictly willing. This is the criterion D1 set-inclusion formulation used below for the stated contingent-plan representation; a different extensive-form representation may induce a different refinement test. □

C.3 Probing

For the sufficient D1 statement in Proposition 3.1, impose

$$\bar{D}_L \geq 0. \tag{C.1}$$

It remains to verify criterion D1 in the contingent-plan representation for $\kappa > 0$. Pointwise state ordering and a common posterior imply that the union of rationalizable seller plans after any price is contained in

$$\mathcal{A}(p) \subseteq \{(x, 0) : x \in [0, 1]\} \cup \{(1, y) : y \in [0, 1]\}, \tag{C.2}$$

where the coordinates are the (L, H) acceptance probabilities. For an undercut $p < q_L$, the wait–probe marginal type $(b, D_L(b; q_L))$ requires, on the first branch, the acceptance probability

$$x^*(p) = \frac{\kappa}{\kappa + (1 - \pi)(q_L - p)}, \tag{C.3}$$

independently of b; every nonmarginal type requires strictly more. Along the marginal curve, the second branch weakly favors higher b because $w_H(b) - w_L(b)$ is weakly increasing. Hence the highest feasible marginal value

$$b_L^0 = \sup\{b \in [\underline{b}, \bar{b}] : D_L(b; q_L) \geq 0\}$$

is D1-permitted and has the largest continuation value among permitted marginal types. Belief on it deters every undercut when $q_L \leq C_L(b_L^0)$.

To verify this inequality under (C.1), define

$$\Xi_L(b) = C_L(b) - w_L(b) + \frac{\kappa}{1 - \pi} = \Gamma_L(b) - d_{S,L} + \frac{\kappa}{1 - \pi}.$$

The competition wedge makes Ξ_L weakly decreasing. If $b_L^0 < \bar{b}$, then $D_L(b_L^0; q_L) = 0$, so

$$C_L(b_L^0) - q_L = \Xi_L(b_L^0) \geq \Xi_L(\bar{b}) = \bar{D}_L \geq 0.$$

If $b_L^0 = \bar{b}$, interiority of the fixed point directly gives $q_L < C_L(\bar{b})$. For an overcut $p > q_L$, no type gains on the first branch of (C.2), since it gives no more L-acceptance than the cheaper equilibrium probe. On the second branch, the required H-acceptance probability is minimized by the highest-value, highest-urgency probing type. Belief on $\bar{b}$ makes H reject below $C_H(\bar{b})$; at or above that price, the top-type inequality in (3.7) rules out a gain even with acceptance in both states. These beliefs cover every off-path price and give the claimed D1 completion.

Full-Menu Refinement

When $\kappa > 0$, the full-menu PBE in Proposition 3.2 likewise admits a D1-compatible completion in the contingent-plan representation. Boundary pricing equates each on-path price to the corresponding seller's posterior continuation value. Global interiority gives positive probing and knockout mass at values arbitrarily close to $\bar{b}$; because C_L and C_H are strictly increasing on a top interval, their conditional means are strictly below their endpoint values. Thus $q_L < C_L(\bar{b})$ and $q_H < C_H(\bar{b})$. For $p < q_L$, the argument leading to (C.3) makes the highest-value wait–probe marginal type $(\bar{b}, D_L(\bar{b}))$ D1-permitted. Belief on that type makes seller L reject because $p < q_L < C_L(\bar{b})$, and pointwise ordering makes seller H reject as well.

For an intermediate price $q_L < p < q_H$, no type gains on the first branch of (C.2). On the second branch, the probe–knockout marginal type $(b, D_H(b))$ requires

$$y^*(p) = \frac{(1-\pi)(p-q_L)}{\pi(r_0 - p)}, \qquad r_0 = w_H(b) + D_H(b) = \frac{q_H - (1-\pi)q_L}{\pi}, \tag{C.4}$$

which is independent of b; types on either side of the boundary require weakly more H-acceptance. The highest-value marginal type is therefore D1-permitted. Belief on $\bar{b}$ makes H reject because $p < q_H < C_H(\bar{b})$, while any L-acceptance cannot generate a gain at a price above q_L. At $p \geq q_H$, even acceptance in both states is weakly worse than the equilibrium knockout. These cases exhaust the nested response set. D1 therefore supports the equilibrium without uniquely selecting it or making implementation belief-independent.

D Welfare Accounting and Numerical Examples

D.1 Payoff incidence

In the boundary-price full-menu equilibrium, the seller receives zero expected rent after a probe in L and after knockout in H. Its gain after knockout in L is $\int [C_H(b) - C_L(b)] h_K(b)\, db$. Hence

$$U_S^D - U_S^B = (1-\pi) M_K \int [C_H(b) - C_L(b)] h_K(b)\, db > 0. \tag{D.1}$$

The early buyer's gain is the integral of its chosen incremental payoff:

$$U_E^D - U_E^B = \int f(b) \left\{ \int_{D_L(b)}^{D_H(b)} V_L(b,d)\, dG(d) + \int_{D_H(b)}^{\bar{d}} V_H(b,d)\, dG(d) \right\} db \geq 0. \tag{D.2}$$

The inequality is strict when a positive mass offers at a strictly positive gain. A later buyer loses only when an early offer is accepted, so the late buyers' aggregate gain from the ban is

$$U^B_{\text{late}} - U^D_{\text{late}} = \mathbb{E}[\mathbf{1}_{A_c}\Lambda_s(b)] \geq 0. \tag{D.3}$$

Combining these equations with (4.3) also gives $W^B - W^D = \mathbb{E}[\mathbf{1}_{A_c}\Lambda_s(b)] - (U^D_E - U^B_E) - (U^D_S - U^B_S)$. The buyer and seller gain in expectation at their own information sets; this does not require that each accepted trade benefit both parties for every realized (b, s).

D.2 The paired welfare examples

Set $(n_L, n_H) = (2, 6)$, $(v_L, v_H) = (.2, .4)$, $\pi = .5$, $d_{S,L} = d_{S,H} = 0$, $\kappa = .02$, and $\bar{d} = 1$. On $[0, 1]$, the urgency distribution is

$$G(d) = 10^{-4}d + (1 - 10^{-4})[1 - (1 - d)^{30}].$$

Compare the common early- and late-buyer distributions

$$\begin{aligned} F^S &= .01U[0,1] + .99\,\text{Beta}(40, 40), \\ F^T &= .01U[0,1] + .99\left[\frac{16}{17}\,\text{Beta}(38, 42) + \frac{1}{17}\,\text{Beta}(9, 1)\right]. \end{aligned}$$

Both means equal .5. The uniform components ensure positive densities. The selected price pairs are approximately (.51098059, .55437879) and (.51561331, .64062633), respectively. These are illustrative distributions, not estimates of an application.

Integration and root verification. The following describes the certificate underlying Table 4.1. For an integer beta distribution, its CDF is a finite polynomial,

$$B_{a,b}(x) = \sum_{j=a}^{a+b-1} \binom{a+b-1}{j} x^j (1-x)^{a+b-1-j}.$$

The mixture CDFs can therefore be enclosed at rational endpoints using outward-rounded arithmetic. For a fixed count n, let

$$A_n(t) = 1 - F(t)^n, \qquad B_n(t) = 1 - F(t)^n - n[1 - F(t)]F(t)^{n-1}.$$

These are the probabilities that at least one and at least two late values exceed t. They are decreasing in t. Together with (A.1), the terminal quantities can be computed from

$$R_{n,v}(b) = v + \int_v^{\bar{b}} B_n(t)\,dt + \mathbf{1}\{b > v\} \int_v^b n[1 - F(t)]F(t)^{n-1}\,dt,$$
$$L_{n,v}(b) = \max\{v - b, 0\} + \int_{\max\{v,b\}}^{\bar{b}} A_n(t)\,dt.$$

The remaining revenue integrand $n(1-F)F^{n-1}$ is unimodal in F, so its endpoint values and its possible interior maximum bound its range on each cell. The calculation uses 24,000 rational cells and 60 decimal digits in Arb. Outer value integrals use Lebesgue–Stieltjes sums: each integrand's interval range is multiplied by the exact enclosed probability mass of that cell. The same procedure bounds the price residuals, their derivatives, the action probabilities, and welfare integrals.

For root certification, use the unnormalized residuals

$$H(q) = \begin{pmatrix} \int [C_L(b) - q_L] m_P(b;q)\,dF(b) \\ \int [C_H(b) - q_H] m_K(b;q)\,dF(b) \end{pmatrix}.$$

Positive offer masses make $H(q) = 0$ equivalent to $T(q) = q$. For a candidate box X with center x, a numerical inverse Y of the central Jacobian, and an interval enclosure $J(X)$ of DH, the Krawczyk operator is

$$K(X) = x - YH(x) + [I - YJ(X)](X - x).$$

Strict inclusion $K(X) \subset \operatorname{int} X$ certifies a root; the nonsingular interval Jacobian also verifies local uniqueness within the box. Initial boxes have centers $(.510980593207608, .554378791664074)$ and $(.515613314493683, .640626332049219)$, and coordinate radii .0004 and .0008, respectively. Table D.1 reports outward enlargements of the resulting root and welfare bounds. This is a local certificate, not a global uniqueness result.

Table D.1: Certified Bounds for the Welfare Examples

	F^S	F^T
q_L	$[0.51087978, 0.51108141]$	$[0.51550714, 0.51571949]$
q_H	$[0.55430282, 0.55445477]$	$[0.64035841, 0.64089426]$
$W^B - W^D$	$[-0.00366207, -0.00339311]$	$[0.00026167, 0.00034508]$
$\det J(X)$	$[0.01303747, 0.01911430]$	$[0.00017958, 0.00033740]$
$\min D_L$, lower bound	0.017569	0.002409
$\min(D_H - D_L)$, lower bound	0.003932	0.102852
$\max D_H$, upper bound	0.598030	0.766282

The strict cutoff margins establish $0 < D_L < D_H < 1$ over the whole value support and the reported root boxes. Positive seller-response margins and $C_s(1) - q_s > 0$ verify on-path acceptance and the endpoint inequalities used in Appendix C. Once an interior fixed point is certified, the equilibrium proof in Appendix B.3 applies directly; the rectangular face test is one way to obtain such a point, and the Krawczyk calculation supplies another.

To reproduce the bounds, run code/certify_ban_welfare_examples.py with Python 3.12 and `python-flint==0.9.0`. The accompanying computational materials contain the script and its machine-readable output. The displayed formulas specify the mathematical objects and enclosures; the Online Appendix is not needed for this certificate.